THE

MATHEMATICAL GAZETTE.

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THE CORRELATION OF THE TEACHING OF MATHEMATICS AND SCIENCE.

REPORT OF A CONFERENCE HELD BY THE MATHEMATICAL ASSOCIATION IN CONJUNCTION WITH THE FEDERATED ASSOCIATIONS OF LONDON NON-PRIMARY TEACHERS.

A CONFERENCE of the Federated Associations of London Non-Primary Teachers, in conjunction with the Mathematical Association, on the subject of the Correlation of the Teaching of Mathematics and Science, was held in the Marlborough Room of the Regent Street Polytechnic on the afternoon of Saturday, November 28th, at 3 o'clock.

The CHAIRMAN, Professor G. H. Bryan, F.R.S. (President of the Mathematical Association), in opening the proceedings, said: This is a joint meeting of the Mathematical Association and certain other bodies to discuss the correlation of Mathematical and Science teaching; and our first item on the programme is an address by Professor Perry on a subject to which we shall listen with the greatest interest.

The more meetings and discussions we have the more we shall further the ends which I believe most of us have at heart. One of those ends is to make the average British citizen more mathematical and, as such, more intelligent, and other ends are to improve the teaching of Science and to raise Great Britain out of the position she has drifted into of being the most unmathematical State on the face of the earth. I hope that this meeting will provide interesting discussions, and that full

use will be made of the present opportunity for expressing opinions. It is desirable that everybody interested in mathematical or science teaching should have some opportunity of discussing such matters. In any case I feel that anything we may be able to do at this meeting in the form of suggestions in furtherance of the good cause we have at heart will be very useful. I now, with great pleasure, call upon Professor Perry to read his address.

Professor J. PERRY, F.R.S., in introducing his subject, said: I should like to say that I have not had immediate contact with boys, as a master, for the last thirty-eight years. Thirty-eight years ago I was in charge of the Physics, and had to assist in the mathematical teaching at Clifton College. I take it that the word science means physical science, and that the term scientist includes the mathematical physicist, but does not include the mathematician who is enlarging the scope of pure mathematics; it includes all men who are teaching mathematics to students who wish to use it in physical investigations. It may be well to restrict the term mathematician to a student of mathematics who is enlarging the scope of pure mathematics. I have been corrected by a senior wrangler, who is himself a mathematical physicist, for saying that a certain other senior wrangler was a mathematician. "No, he is a mathematical physicist; he would not be pleased to hear himself called a mathematician." I suppose we must consider the cases of: (1) The training of all children, the citizen of the future being supposed to be taught science for various good reasons: one, that he may take an interest in those things which are more important in transforming the world than all others; another, that he may learn scientific method, which will cause all his actions to be more efficient. Every man who has done important things in history was distinguished by his having scientific method, not perhaps gained in our modern way. We want the average man to have it now. (2) The training of the boy or man who intends to enter a profession of applied science. (3) The training of average men who intend to teach mathematics and science. (4) The training of a few mathematicians and scientists who may possibly become eminent, and may or may not become teachers. I do not think that the early training of any of this class should differ from that of the others.

When I was told that I had the honour to open this discussion I forgot that the word "teaching" was in the title. I meditated on the quite different views taken by the mathematician and the scientist about a scientific investigation. The one insists upon mathematical rigour in every part of the reasoning. As I believe that there is hardly one rigorous proof of anything,

I had better say that he is orthodox. The other says that tedious and apparently exact mathematical proofs are not only unnecessary, but perhaps harmful if there are physical proofs. In investigations in heat, hydrodynamics, elasticity, electromagnetics, etc., if we follow orthodox methods we can take up only easy questions; if we use Heaviside's operators, including such unorthodox things as divergent series, we can investigate questions which the orthodox person would not dream of touching. It is easy to prove that a result is correct, and that no other correct result is possible. But, alas! in the present state of knowledge Mr. Heaviside himself cannot give an orthodox proof that the use of his operators is legitimate. And I have no doubt many of you know the following quotation from the preface to the second edition of Lord Rayleigh's "Sound": "In the mathematical investigations I have usually employed such methods as present themselves naturally to a physicist. The pure mathematician will complain, and (it must be confessed) sometimes with justice, of deficient rigour. But to this question there are two sides. For, however important it may be to maintain a uniformly high standard in pure mathematics, the physicist may occasionally do well to rest content with arguments which are fairly satisfactory and conclusive from his point of view. To his mind, exercised in a different order of ideas, the more severe procedure of the pure mathematician may appear, not more, but less demonstrative."

This opposition of the mathematician and scientist can hardly be avoided, but it need not create bad feeling. The scientist has the greatest admiration for the mathematician, but he asks for more sympathy and no dictation at all. As practically all branches of mathematics have sprung from the study of science, might not a capable mathematician see some prospect of glory in devoting his life to a proof that Heaviside's operators are legitimate? We know that they are, but we have no orthodox proof. It is to be remembered that in the days of great scientific development, the days of Leibnitz and the Bernoullis, the days of Lagrange, the days of Fourier, powerful methods of analysis were used which were not rigorously proved to be legitimate until long afterwards. As to teaching, are we not agreed that mathematical chairs and fellowships should be so well endowed that no mathematician shall be compelled against his will to attempt to be anything but a mathematician? At present we compel him to attempt pedagogy.

It is always forgotten that those very qualities which cause a man to become a great investigator in pure mathematics cause him to be unsympathetic with the student who is not also a pure mathematician. How many colleges are there with only one teacher of mathematics? And it is almost always

the case that he is a mere mathematician, and therefore quite incapable of teaching the average student, whom he looks upon as a stupid person. I know of colleges where mathematicians of the highest rank are put to teach mechanics to average boys who are beginning the subject. It is one of the most harmful of things for students known to me. It is probably harmful to the mathematicians also; but the opinion of a man like Sir A. Greenhill ought to be asked on this point. It seems to me like harnessing Pegasus to a common wagon.

It is only another example of how a clever specialist gets credit for being an all-round great man, able to give the best advice on subjects which he has never thought about. In England we think that if a man is a great mathematician, his voice in politics is important; if he is a great chemist, his opinion on women's suffrage is valuable.

Mathematicians attempt also to act as examiners, or they control examinations. They even attempt to write text-books upon science. If you open such a book, say on hydrostatics, it is all easy pure mathematics; easy examples of integration, for example, disguised by the use of such words as "pressure" and "depth." And yet when a simple algebra problem is set about apples or the hands of a clock, nobody supposes the book a treatise on gardening or horology. A treatise by a mathematician is not necessarily scientific because it contains words like mass, temperature, and electromotive force.

The scientist needs teaching which will give him such a command of mathematical methods that he can apply them in any new problem with ease and certainty. Whether or not he has studied rigorous proofs that these methods are legitimate, he certainly must gain familiarity with them by actual use. I believe that some use, and even much use, ought almost always to come before the proofs, and in some cases the proofs need not be studied at all if it is felt that their study does not tend to utility. To take familiar examples: a student ought to use logarithms long before he approaches the proof of the exponential theorem. He ought to use Taylor's theorem long before he studies what the mathematician might call a rigorous proof of it. He ought to develop arbitrary functions in Fourier series long before he studies an orthodox proof that such development is possible. He ought to work many problems involving the use of spherical harmonics or Bessel functions long before he studies these things in the way that a mathematician insists upon as necessary.

An old and distinguished naval officer wanted to enter a great naval college merely to rub up his knowledge of the reasons underlying the methods of the navigating lieutenant. I could have given him what he wanted easily in a month. He was

told by the principal that he must begin Euclid's geometry again, and that he must study mathematics for years. The principal of the college was a mathematician.

Again, there are always separate examinations in geometry, algebra, trigonometry, and mechanics, because the mathematician insists upon it. A good teacher will and does teach each of these through the others; in fact, they are all one subject. But, unfortunately, the separate examination system causes some of them to be taught out of all proportion to their educational or other utility. We have all, probably, quite different views as to their real utility. Thus I consider almost the whole of deductive geometry to be a university study, or even perhaps a post-graduate study. It is certainly not needed in the elementary study of science or in the study of those parts of mathematics which need to be known to the young scientist. I think that most scientists hold my views, but probably no mathematician does. I take it that the method of study into which Newton was forced, became, because of Newton, the favourite English mathematical study, and we know that it kept English mathematicians back for a hundred years. In the shape of elementary deductive geometry it is keeping back every schoolboy now. I do not believe that any average boy, indeed I do not believe that any boy under sixteen, understands the very first theorem in the first book of Euclid. At all events, I suppose that all scientists are agreed in saying that, however mathematical reasoning may start or have started, it becomes abstract as it develops, and so the average boy is never taught mathematics, and cannot be taught. He may, however, be taught mathematical method as any other physical science is taught, say mechanics or heat—with experiment and common-sense reasoning. But we must not allow the mathematicians to interfere. To them the study of geometry is really logic, and it follows that there must be a division of the subject into propositions. We have long given up the use of catechisms in teaching history and science; a series of propositions in geometry is just as foolish, whether we follow Euclid or not.

I mention these differences in the points of view because mathematicians do attempt to teach and to examine and to control examiners. In my presidential address to the Physical Society in 1907, I pointed out that the peculiar conditions of one examination at one British University had led to the creation of 90 per cent. of elementary algebra and trigonometry in Great Britain, this unnecessary 90 per cent. being as complex and tricky as it was possible to make it. Until all this sort of thing is done away with, the marriage of mathematics and science seems to me like that of December and May—the marriage of a man of seventy with old bachelor

habits to a bright young virgin of seventeen. The teaching of mathematics still follows the old lines; we are compelled by examination systems to refrain from suggesting drastic reform, and we can only tinker with it. The success of what is called *practical mathematics* in every technical college and school in the country, in interesting hundreds of thousands of students more than they are interested in any other subject, proves that it is possible to remove the incubus which has stupefied so many generations of schoolboys.

Just as in trigonometry the labour-saving rules of the surveyor have been dignified into a quite unnecessary subject called "The Solution of Triangles," so we have labour-saving rules in mechanics and kinetics and other parts of mathematical physics. A chapter of a text-book is devoted to the proof of some of these rules, and there are thirty exercises, each of which a student can work out in about twenty minutes. This is very handy for examination purposes, when there is a very limited time, but it is pernicious in all other ways. Now why do I say that it is pernicious? First, because it tends to make a man think too much of labour-saving rules, of formulæ. Fundamental principles are neglected. Secondly, a man who is not constantly teaching forgets these rules quite quickly. If he desires to work a problem, he thinks of how once he could do it at sight, but he has now forgotten all about it. He remembers how much trouble the original study gave him; he cannot again face such a worry, and he is disgusted with himself and his mathematical training.

Now, my students are able to tackle difficult new problems in statics or kinetics, and they can be certain that their answers are right, but they must get plenty of time, perhaps ten times as long in some cases. I give them a few principles only, such as Newton's one great law of motion, sometimes called D'Alembert's Principle, and I let them see what it becomes in a number of cases. But whatever illustrations I may give, I always point out that it is merely an application of the one general principle. They get so thoroughly acquainted with this as a general principle that they can apply it readily (exercising their well-developed common sense) to exceedingly difficult-looking problems, such as may turn up, and furthermore they really cannot forget it. All through our teaching we give all sorts of information which a student can at any time find in books if he knows how to use books; things not essential to an educational study. When we give an investigation we do not sufficiently impress a student with the fact that it is merely an illustration, an algebraic development of some simple general principle. He learns it detached from all other investigations, and he has

therefore twenty times as much to remember as he need have, so of course he forgets it all. This evil is easily traceable to the influence of the mathematician on examinations, and therefore on teaching, at all events in such a subject as dynamics. And because the method is orthodox in dynamics, everybody brought up in that way—the science teacher, for example—is induced to neglect common sense equally in the teaching of other subjects. To keep ourselves right, to prevent fashion and habit or bad example from leading us astray, we ought to stick to one great principle—our object is not to convey information, or a thousand labour-saving rules, our object is to teach scientific method!

I have studied the suggestions of the three committees of the Mathematical Association—one on the teaching of arithmetic and algebra, one on the teaching of mechanics, and one on advanced school mathematics. The first of these will be found reprinted in the report of the British Association Committee (1906, York) "On studies most suitable for elementary schools," which I would advise everybody to read. It contains also the report on the teaching of mathematics by a committee of the Institution of Civil Engineers, and a statement of the views of the Board of Education on the teaching of elementary mathematics. All these reports appear to me to be excellent.

Will you forgive me for saying that, all so excellent as the syllabuses of the Mathematical Association are, they seem to me to possess too much of the orthodox mathematical spirit? There is too much hankering after a kind of logical perfection which is impossible in the teaching of the average boy. I am afraid that what seems to you simple is to him complex, and what seems to you complex is to him quite simple. As a result, you have not made his studies as interesting to him as you might, and whatever is uninteresting to him is uneducational. Besides, you seem to have forgotten that practically all that is bad in school teaching comes from our having separate courses of study, each with its separate examination.

In arithmetic, mere number seems to you simple. To the average man this is an abstraction not perhaps so incomprehensible as abstract ideas of time and space, but never studied even when he shepherds his family to the seaside, and he says he has twenty-four parcels, a perambulator being one and a hat-box one.

It used to be that a boy had almost nothing explained to him; he did his multiplication, etc., mechanically. Now we are always explaining and philosophising. Either method is bad. A good method can be made up out of a mixture of the two. The philosophy of nearly all things, just as in swimming and playing whist, ought to follow the doing rather than to

precede it. Some teachers maintain that the algebraic method of detached coefficients is valuable for the comprehension of arithmetical multiplication. I would as soon recommend the study of English through Latin and the use of steam hammers for cracking nuts.

A boy thinks of quantity, and the sooner you get him on to algebraic arithmetic the better. 6 cows $\times 3 = 18$ cows, or 6 cows $\div 3 = 2$ cows; he has no difficulty in these. Again, in finding the area of a rectangle he has no difficulty in understanding 6 inches $\times 3$ inches $= 18$ square inches. Again, we never find that he has a difficulty in understanding work to be

$$6 \text{ pounds} \times 3 \text{ feet} = 18 \text{ foot pounds},$$

and he at once agrees to $6b \times 3a = 18ab$.

But in the abstract all these are just as complex as

$$6 \text{ tables} \times 3 \text{ chairs} = 18 \text{ chair tables}.$$

He has no difficulty with anything of this kind if he knows the nature of the product. He says that to talk of 18 chair tables is nonsense, because nobody has ever shown him that a sensible meaning might be given to a chair table. Now I believe that $6 \times 3 = 18$ really means to him 6 feet or shillings $\times 3 = 18$ feet or shillings, and the more you reason as if these were numbers in the abstract the more you confuse him.

In your mechanics syllabus you object to

$$\frac{\text{weight}}{g} = \frac{\text{Force}}{\text{acceleration}}$$

and you insist on using

$$\frac{\text{Force}}{\text{weight}} = \frac{\text{acceleration}}{g},$$

but either is logical and simple to a boy. Even in that same syllabus you allow a boy to divide quantities of different kinds by one another, and this is right, for a boy will see no difficulty. A man swims 160 yards in two minutes. Of course the average boy says at once that the speed is 80 yards per minute. He has divided 160 yards by 2 minutes.

It is astonishing how much a boy can take in intelligently if you will only not worry him by telling him how difficult he ought to find it. Thus when a weight of 3 lb. is lifted 2 ft. the work done is 6 ft. lb. When a force of 3 lb. acts at a distance of 2 ft. from an axis, the moment of the force is 6 lb. ft. Now this is a different thing altogether, and yet a boy has no difficulty in thinking about them both. All this leads to the idea that all parts of elementary mathematics ought to be taught along with and through science, and by the same master.

As I said at Johannesburg, "Children should in their play be accustomed to measurement; playing at keeping shop, selling things to each other by weight and measurement, paying for things in actual money. Measurement of things with calipers and scales would accustom boys of eight to the use of decimals. Things requiring memory can be learnt only in early youth; weights and measures, the multiplication table, languages, plays involving spelling, the repetition of poetry, surely these are the things that ought to occupy children when very young. The reasoning powers will gradually develop in a healthy way if only they are not forced; powers of memory and of observation are at their best, and only need guidance to equip the boy with stores of classified facts. Thus, too, this early time is the time to let a child find out for himself that if he can read he can use story books, and if the people about him are fond of reading he is certain to become fond of reading also.

". . . And after a boy reaches the age of ten he may sketch and draw plans of his schoolhouse and the roads or streets about it; he soon can use a map when walking or cycling, and he knows that maps may be of different scales. I would lead him up to the vector subject of geometry only slowly, through maps useful to himself, through problems on heights and distances and the like. Do you remember a story called 'Sandford and Merton,' where a Mr. Barlow taught two boys by methods surely the most interesting and delightful? It is between the years of nine and thirteen that boys need a Mr. Barlow, and I can imagine one Mr. Barlow being able to look after ten boys, and perhaps more. There were no formal courses of study in many subjects such as exist in schools, each in its water-tight compartment. One night the boys are interested in the stars; that night he tells them about the stars, and lets them look through a telescope. He gives them stars and solar system just so long as they are interested. He uses a globe as well as mere maps in teaching them geography, but the soul-destroying idea of a course of study on 'the use of the globes' does not commend itself to him. By pleasant experience they learn that there are rules about levers and parallel forces, and with simple apparatus he illustrates these rules. This is the time, say at the ages of eleven to thirteen, when boys ought to have a course of experimental science, weighing and measuring accurately, learning the rules of mensuration, taking specific gravities, learning something of barometers and thermometers, of magnetism and currents of electricity. People do not seem to know how cheaply and easily quite a lot of interesting apparatus may be rigged up by boys. They make electric machines and Leyden jars out of bottles bought cheaply from grocers. They can arrange an electric telegraph,

and after they have learnt the code they can signal all sorts of messages to each other in all sorts of ways, not merely electrically, educating the senses of hearing and sight and touch. Their hands are educated by the use of tools, and so they learn all sorts of properties of materials. Thus I advocate a course of elementary science which, to be followed by a boy, involves the continual use of computation of all kinds; which keeps a boy interested; which satisfies his voracious curiosity; which is first qualitative, and may be made just as quantitative as the boy and teacher please to make it."

In the perfect school all form-masters below the middle fourth (I wish I could say fifth) ought to teach mathematics and natural science as well as Latin and English and French and German and geography and history and many other things. It may be objected that if in one form (less than half the present size) a master teaches many subjects, the boy best in Latin may be worst in algebra, and there will therefore be great differences in the calibre.

I know from experience that, at all events in practical work of any kind, and I believe in all the work of such a form, it is actually good educationally to have considerable difference in the calibre of the boys. If a group in charge of any investigation consists of three boys of different capabilities in that subject and of other kinds of differences in other subjects, the mutual education going on is very noticeable, it is very valuable.

Some of you will think my ideal system to be impossible, but as I contemplate much better salaries and twice as many masters, a little reflection will show that it is quite possible in the future. It is possible even now to have it that from top to bottom of the school there shall not be a master who is only a specialist, who is a mere mathematician, or who is merely a classical scholar or a mere chemist or a mere anything. The attainments of all masters shall be such that no one will ignorantly sneer at any subject studied at the school. In the upper forms I presume that the present system cannot yet be greatly changed, but just as the geography and history of Greece and the Greek and Roman Empires ought to be taught by the form-masters along with literature and grammar, so I consider that geography generally, and history generally, ought to be taught by them. I do not mean that elaborate courses ought to be given, but lessons may be given incidentally. I know that some intelligent masters do this at present, but all the masters ought to have sufficient culture to do it.

The aim ought to be, not to make a boy a geographer or a historian, but to give him such a general interest in all

these subjects that he will read things for himself. We ought not to aim at producing finished products either at school or the university. We ought to try to produce learners.

All form-masters ought not only to know at least two subjects well, but to have such an acquaintance with all the other subjects taught at a school as to be able to talk intelligently with their boys on all these subjects, although they may not be able to teach. If my ideal is impossible now, at all events we can work towards it.

I take it that the specialist, if he teaches at all, ought to teach university, or rather post-graduate, students. Until a man takes his degree his teachers ought to be men of general intelligence. When a man has become a Greek scholar he may, if he sees fit, neglect all other studies; he may neglect Greek archæology even; when a man becomes a mathematician he may neglect all applications of mathematics if he thinks it better for his pure mathematics investigations.

Especially is it important that every master of a school should consider himself responsible for English composition. If a boy writes a description of anything he has done in a laboratory or elsewhere, it ought to be an exercise in English. No sensible teacher now gives marks for lecture notes, but rough laboratory notes ought to be accompanied by clear accounts of what the boy has done, illustrated with good sketches and squared-paper curves.

For the present it is too much to expect that higher form classical-side masters will be able to teach physical science, so here, and also in higher modern-side forms, there will continue to be the set or block system with other masters than the form-masters. But each such other master ought to teach several subjects. In critising this it must be remembered that no boy in a school ever reaches the level in mathematics which requires a mathematical specialist to teach him, and it is the same in physics and chemistry.

I was a pupil of the famous Dr. Andrews, of carbonic acid fame. He was supposed to teach only chemistry, but in truth he taught chemistry and physics, and we learnt much more about each of these subjects than if we had had two teachers of two detached subjects.

I remember that when I was ten or eleven years of age I had a master who taught me geography and history and grammar and literature in a delightful jumble; but his lessons were educational. He would suddenly tell us on Friday that each of us was to write him a letter for Monday morning on any subject whatsoever. Observe, we only did any particular kind of exercise occasionally. If we had had to do it every week it would have bored us and him. He gave us four, and only four,

delightful lessons on philology. If he had recently been to see Barry Sullivan or G. V. Brooke acting Hamlet or Richard III. or Macbeth or Othello, he read and made us read extracts from these plays, and he made history and literature and geography and other things that boys call "subjects," fairly lovely to us. In the science lessons, one day he would have for us a great basket of buttercups or daisies, and each of us would pull his specimen to pieces as he did, and we followed him in all his explanations of what we saw. Another day he brought in a capillary glass tube, and he blew a bulb and filled the thing with mercury, and marked on it the freezing and boiling points of water, and taught us the use of a thermometer. If he had had modern-school laboratory appliances I feel sure that he would have followed the methods of Dr. Armstrong.

After a boy has made up, by the use of objects, a multiplication table, I feel sure that you ought to leave mere number and get to quantity, so that he ought to use decimals quite early. To teach him the use of decimals and to educate his hand and eye and judgment, you allow him to measure things. But how do you do it? Often in the most uninteresting way! He is made to measure a certain length, to weigh an object, etc., and his answers are compared with the real length and weight, etc. Contrast this with the following exercise. He is given a block of iron, and he measures its length, 3·27 inches, breadth, 2·63 inches, thickness, 1·95 inches. He finds its volume to be 16·77 cubic inches. He readily uses contracted methods. Why will you try to stop him? He is given a cube 1 inch in edge of the same kind of iron. He takes it to the scales, and finds it weighs 0·26 lb., so he computes the weight of his block to be 4·36 lb. He now goes and weighs it, and is delighted. Do you see where the difference comes in, and how interesting it is to find his computation agreeing with reality? And that same piece of iron was allowed to displace water in a vessel, and the displaced volume was found experimentally to be nearly 16·77 cubic inches. He now takes an irregularly shaped piece of iron, finds its volume by displacement, computes its weight, and weighs it by the scales. He sees at once how it is that perfect agreement cannot be expected.

Common-sense explanation accompanying experiment ought to be the rule. Do not teach abstract geometry at all; teach mensuration with the help of arithmetic and algebra and weighing and measuring. Do not be afraid to introduce a boy early to sines, cosines, and tangents, and the calculation and actual measurement by surveying, etc., of heights and distances. Boys are intensely interested in such work, and it is educational. But how awfully dull it may be made!

It astonishes me to see how little comprehension there seems to be of the proposals made by the British Association committees. We recommended experimental geometry with common-sense reasoning, and everybody seems to think that we asked for a babyish use of rulers and compasses following a series of propositions. We asked for interesting work in weighing and measuring, and care is taken that all such work shall be made as uninteresting as possible. We recommended some work with graphs on squared paper, and some teachers do nothing but graphs, and there are dozens of school books to help on the craze. It reminds me of a friend who does so many things with cement that I almost believe that if he wanted a new umbrella he would make it of cement. Some teachers think that squared paper was invented merely to illustrate the solution of certain simultaneous or quadratic equations.

I do not say that some teachers are carrying our reforms too far; I say that they do not at all comprehend the nature of the reforms suggested by the British Association and other committees. In the higher school work we ask for a rapid advance to the calculus and the use of calculus methods in teaching kinetics and physics, and this is being carried out in such a way that all algebraic work is getting to be quite inaccurate and slipshod. Surely there can be just as good mathematical drill in using the calculus as in the usual tricky algebra and trigonometry. Indeed, the drill is on exactly the same kinds of algebraic expressions. It used to be that if a man was doing calculus there was no need to inculcate carefulness and accuracy, because he had already been well drilled. Accuracy in algebra, accuracy in measuring, each of these is as essential as the other to the success of our reforms. The surprising thing is that many teachers seem to have no individuality, no originality, nor even the power to think for themselves at all. The principle on which we work seems to us quite simple, and they think they are following it; but really they no more understand it than the average Mohammedan understands the Christian faith.

A fixed ritual will be looked upon as a curse by the good teachers of the future, but it almost seems that at present we must impose some system of teaching so complete in every detail that any teacher can follow it exactly. We must not only tell a teacher to make his students' experimental work interesting, but we must tell him exactly how to do this. Dr. Armstrong's valuable addresses on the teaching of science seem to teachers to be masses of vague generalities, and if he is to succeed I feel sure that he must do what his soul abhors, and describe a lot of experimental discovery work and

exactly how it is to be put before a class of boys. When he says that a mere knowledge of facts is unimportant—we must cultivate scientific method—how few teachers there are who see what he is driving at. Yet he is only enunciating a principle which has been advocated by all educationists—Milton, for example, and Herbert Spencer—and has been acted upon by all good teachers who had freedom to teach as they pleased.

Dr. Armstrong deals more particularly with the very young boy. His work is experimental, with no difficult computation, with common-sense reasoning. To the age of fourteen, when, of course, every boy ought to be able to use easy calculus methods, his work is still mainly experimental. He has become more accurate in his algebra work and in his weighing and measuring. He has taken certain things in hydrostatics, pneumatics, chemistry, heat, and even electromagnetics as parts of one general elementary study. At the age of twelve he had experimented on levers and friction in machines, and knew something about mechanical and other forms of energy and centrifugal force. Later he experimented upon the triangle of forces and resolution of forces, using graphics, and learning of sines and cosines about the same time as he was studying heights and distances. But he had broken wires and beams long before. After he began kinetics he quickly had a clear idea of speed and of the fundamental notion of the calculus. Only at sixteen does he begin to reason about the addition, subtraction, multiplication, and differentiation of vectors generally. By means of water flowing into and out of vessels, with the conical pendulum and other apparatus, he learns exactly what is meant by "force is rate of communication or change of momentum," momentum being a vector quantity, and he sees by many illustrations that with this fact well understood he has the easy means of solving numerous problems which now seem hopelessly difficult to many B.Sc.'s. But before this he has learnt of the analogies between motion of translation and angular motion of rigid bodies, and he now sees the analogy between constant centripetal force on a particle and constant torque about an axis at right angles to the axis of a rotating body. Throughout all this I have been speaking of the average boy. At sixteen or seventeen he has a good quantitative knowledge of physical science; his mathematical tools are familiar to him, and he can apply them at once on any new problem. His common sense is in a wonderful state of development. He was fond of reading when young, and he knows now how to use books. It would be waste of time to teach him 95 per cent. of the mere facts which he can find in books by himself. He is not going to give up this kind of study

when he leaves school; he will go on educating himself by means of it until he dies, for he loves it and cannot forget it. Just imagine all this in combination with his other studies in literature and history. Surely he will make a fine citizen who, when he invests his money or gives his vote at an election, or when he sits on councils and committees or on the magistrates' bench, will feel that he can act on opinions of his own, and that he is not at the mercy of impostors.

In conclusion, may I say that if any man thinks me to be a nuisance, that I am doing harm in advocating changes, I do not mind his saying so if he gives reasons for his belief? No personal friend of mine need fear diminution of my friendly feeling through his doing what he thinks to be his duty. I hope that nobody will feel unfriendly because I think it my duty to express myself clearly.

After all, nobody here can think that I am referring to him when I say that a mere specialist cannot teach. His presence at this discussion is in itself an evidence that he is not a one-sided man. It used to be that mathematics, physics, chemistry, etc., were completely detached from one another and from all other subjects at a public school. But I can say now with confidence that every man in this room who has had more than one year's experience as a master in a public school of these days, must already have lost that detached attitude if he ever had it.

All I ask is that this action, of which we are all conscious, shall be allowed to go on unchecked. Until salaries are doubled and forms halved in their numbers of boys there will be things very open to criticism. Our weaker brethren, in their honest endeavours to be interesting, will muddle things up. Good systems will be established, but the unbelieving teacher or the ignorant one or the over-worked one will make stupid rituals of them. Happily, there are many good teachers who are fond of teaching, and go on teaching, although they might make much more money otherwise. It is our duty to make the public believe in these men and see that they are better paid, for that is the only way in which a greater proportion of able men are to be induced to become teachers.

The CHAIRMAN said: Ladies and Gentlemen, it has been of great interest to me to come up and listen to this able address by my great friend and, may I say, my great enemy, Prof. Perry; for one's greatest friends are those with whom one always enjoys a quarrel. It is always a delight to try and pick a quarrel with Prof. Perry. He is such an excellent opponent. I am afraid he is too sensitive in regard to what a certain distinguished Senior Wrangler said as to who is a mathematician and who a mathematical physicist. I do not think he need attach very much

importance to that distinction. I have always claimed myself to be a mathematician and not a physicist, because I have always thought that a mathematician was a man who, when he wants to find anything out, uses his brains for that purpose, whereas a physicist—(loud laughter)—when he wants to find out anything, resorts to experiment. I have always thought it kind of the physicist to call a man a mathematical physicist when he uses his brain to find out some result and leaves other people to do the experimenting. On the other hand, the physicist becomes a mathematician when he uses his brain over interpreting his experiments.

Prof. PERRY, interposing, said he had referred to Prof. Larmor, Secretary of the Royal Society.

The CHAIRMAN (continuing) said: Prof. Perry need not have explained, because I guessed, long before now, to whom he referred. I believe that Prof. Perry is as much of and as good a mathematician as any one we have going. I hope he won't consider this an insult. (Laughter.) My own opinion is that, as the address now reads, Prof. Perry is too hard on the individual who is doing research work in Pure Mathematics. I know that many of the men who are doing that kind of research are as keen on Prof. Perry's side as anybody. They know the difficulty of Mathematics. It was recently said by Mr. Lloyd-George at Bangor that what we want at our Universities is not merely teachers, but explorers who shall explore the undiscovered regions of Science, the mapless continents, and the chartless oceans of the unknown. I believe that the real explorer in Pure Mathematics is perfectly in sympathy with everything that can be done to remove difficulties from the beginner, and to carry him higher in his studies. We shall never get explorers so long as we keep them walking a measured mile over a well-trodden syllabus. They will never get as far as the unknown country, nor will they have any idea of what it is like. What we want to do is to bring people up by express train to the front, to get them as near the new land as possible. Of course we must teach them the methods of exploring, but let them do a few obstacle races when climbing over particular parts near their destination, and carry them out by railway train over the uninteresting cultivated regions, instead of making them all tramp over the same measured syllabus.

The people that are against us are those whose idea of research consists in inventing "pretty" problems which take them six hours and thirty sheets of paper to work out, and then setting them in an examination paper headed "Algebra" or "Statics and Dynamics" for which three hours is allowed. (Applause.) Those are the people who retard progress, and I say that Prof. Perry is exactly right when he says that

we must bring boys up to the forefront. What is the use of our spending a lot of time in teaching them methods of finding G.C.M.'s and L.C.M.'s, which they will never have to use for any purpose whatever, when we want to bring them up to the forefront, and, if possible, get fresh explorers to work at that Theory of Groups which is being explored in America, and which, in the course of time, must lead to important applications. I believe Prof. Perry will get some very good material for applications out of that theory, when explorers have first made their discoveries, and when the colonists have been over it and surveyed it, and discovered means for cultivating it. We do not know anything about its practical applications now. I hold the view that the mathematician who has the advancement of mathematical research at heart should not be debarred from teaching and from setting examination papers if he wishes to do so. As regards the mathematician having an all-important voice in politics or anything else, I am afraid I take the opposite view entirely from that held by Prof. Perry. What I complain of is, that politicians and people of that sort are not sufficiently mathematical, and that in this country the mathematician is an unknown being who is regarded as only fit to shut himself up, to amuse himself with X's and Y's, and to live on bread and water. (Laughter.) What do the mathematicians say in answer to Prof. Perry's recommendations for early advancement in Mathematics to meet the needs of physical research? Dr. Carslaw has written a work on Fourier's Theorem, and if you want to learn the whole of the why and wherefore of that theorem, you will find that it requires a man to devote years of study to the subject. That is the answer to Prof. Perry. In other words, it is quite impossible for any one to give a rigorous proof of Fourier's Theorem without being a specialist. What is the answer that the mathematician has to give regarding deductive geometry? Look at all the literature that is being published by the American and Continental mathematical journals on the foundations of Geometry. These foundations are being studied by mathematicians of the highest reputation, and by explorers in the front rank of research in Pure Mathematics. We have got quite past the time when we thought Euclid's proofs were conclusive. As regards his statement that ninety per cent. of the Algebra and Trigonometry is useless, I can only refer to my early days when Algebra simply made me disgusted with Mathematics, and turned me from a pure mathematician into an applied mathematician. Again, when Prof. Perry refers to the undue importance attached to certain formulae for the solution of triangles, I claim that it has been of great use to me to have taken part in examination work, because I have

gained some experience of the results to which the excessive use of these so-called "labour-saving formulae" may lead. If I set a simple question in which candidates are asked to find the area of a right angled triangle, say the area cut off from a rectangular field by a footpath crossing one corner, some of them first find one of the unknown sides from

$$\frac{a}{\sin A} = \frac{b}{\sin 90°},$$

always taking care to put in the sin 90°. They then find the remaining side from a similar formula, also carefully mentioning sin 90°. They do not use logarithms as a rule, but calculate the sides by long multiplication or division going to eight significant figures, if four have been given. They then say

$$\text{Area} = \sqrt{\{s(s-a)(s-b)(s-c)\}}.$$ (Loud laughter.)

They multiply this out by long multiplication and then extract the square root by the long method.

(A voice—"Does he get it right?")

What he generally does in the end is to put the decimal point in the wrong place, and so he gets an answer of (we will say) 374,972,831 square miles, 121 square yards, 7 square feet, $11\frac{2}{5}\frac{7}{0}\frac{3}{0}\frac{2}{0}\frac{1}{0}$ square inches, or something of that sort. (Laughter.)

Now, as regards making the study more interesting. Prof. Perry would, I think, agree with me that there is too much teaching and too little learning. That is at the bottom of the mischief. Another point in this connection, it seems to me, is that if we had not got to face examinations we need have no syllabuses at all. There are a good many things to be said both for and against examinations. The plea that I would put forward in favour of the examination system is that it to a certain extent compels a man to pay attention to his English composition, and to that power of expressing his ideas clearly in writing, on which Prof. Perry, and I in equal measure, think great stress ought to be laid. I suppose that the trouble we all have in examination work is due to a want of trust between the teacher and the examiner. I myself have had to examine at one college where the professor was not supposed to know who the examiner was, and I must say that the position was extremely difficult. The college authorities sent what they believed to be a definite syllabus, but one could not tell from that what were their methods of teaching. I believe that some of my questions were not liked, and I felt that great difficulty had arisen from my not knowing what methods were being used by the teachers. Thus I had to set questions which probably involved different methods from those adopted. Yet the boys managed to show intelligence in their answers, and did well. But it would have been

better if I had been able to ask the professor to send up specimen questions, and to inquire what methods had been used in teaching parts of the work on which I proposed setting questions. Again, Prof. Perry would like us to use

$$\frac{\text{weight}}{g} = \frac{\text{force}}{\text{acceleration}},$$

and he could not possibly object to the boys using force $\times g =$ weight $\times$ acceleration. I say that a teacher should certainly allow a boy to use methods of that kind, if he sees that the boy can do so intelligently. Let him develop the intelligence of the boy as far as he possibly can on those lines. I believe that Prof. Perry would do so successfully. But if you include such methods in your syllabus you will have the examiner setting questions on them, and all the teachers following like a flock of sheep and telling their boys that they have got to get them up for the examination. The boys may be allowed to use such methods, the examiner should be prevented from forcing them to do so.

As regards the question whether all masters ought to teach everything, I think there is far too much of that going on at the present time. I believe we ought to get men who are sufficiently specialists in teaching a subject to know when to stop worrying a boy about the preliminary details, and to try to develop the boy's ideas as far as the power of the boy goes. I do not see how a man who has had a smattering of Latin, Greek, Mathematics, History, Philosophy, Welsh, Chemistry, Music, Nature Study, and such subjects, can be expected to inspire the average schoolboy. I think that is the kind of knowledge which leads a teacher to tell a boy that "It must be so, because it is in the book, or in the syllabus." But at the same time we do want to encourage the boys to become intelligent and to think intelligently about things. I believe there is hardly any point on which Prof. Perry and I are not in hearty agreement; but there is one further point about which I should like to speak. You will see in an article of mine which is to appear shortly in the *School World* that I recommend that babes and sucklings should be allowed to play with slide-rules, and that before learning their addition and multiplication tables they should know the use of the addition and multiplication slide-rules, being taught to use logarithms as soon as they come to any lengthy multiplications.

We are all much indebted to Prof. Perry for having come here and spoken to us. He has for a great many years been trying to ram a lot of things into our heads, which I believe will do a great deal of good, and I therefore consider we are under a great obligation to him for the interest he has taken in the present proceedings. (Applause.)

Mr. GODFREY: Doubtless a good many of those present in this room are mathematicians. We have to admit that many of the hard things which Prof. Perry says of us are only too true. I dare say that we shall make something of a fight for deductive geometry; but I, for one, am not prepared to defend the presence in the curriculum of Mathematics if divorced from the practical subjects—Physics, Engineering, and what not—to which Mathematics should be a handmaid.

I hold that Mathematics should be taught as a tool. If regarded in this light, elementary mathematics gains rather than loses in dignity. The advanced pure mathematician has a position in the world of study to which everyone will defer; the elementary pure mathematician is neither useful nor impressive. There is something petty and "classroomy" about elementary algebra, if taught without any outlook upon the world of scientific achievement to which algebra may be a key. Many of us mathematicians feel that we must get into touch with the concrete studies which our mathematics should subserve. This leads us to welcome this discussion on the means of correlating Mathematics and Physics.

Geometry stands perhaps in a slightly different position from that of Algebra, Arithmetic, and so forth. Geometry is in itself a branch of physical science. The geometrical properties of bodies are properties of matter as much as are the physical properties. Geometry is not merely an ancillary study. Much might be said about the teaching of Geometry, but I suppose that this would be remote from the object of the present discussion. I shall therefore content myself with expressing the hope that the convenience of examiners will not so far prevail over the interest of education as to enforce the adoption of a fixed sequence.

Even in Geometry a man's teaching will improve as his interests grow wider. With regard to other branches, I would go further; I would assert that the teacher ought constantly to bear in mind the physical sciences to which Mathematics are to be applied.

At this point it is convenient to refer to a matter in which Prof. Perry's doctrines have excited some alarm. I fancy that a good many people have the impression that Prof. Perry would have Mathematics taught *incidentally*—as a digression. This may not be really what he intends; if not, I think it would be helpful if he were to clear up this point. For myself, I have no faith in teaching a subject incidentally. To teach Algebra we must have definite Algebra lessons in a definite course. Incidental teaching may be successful in the hands of a single teacher here and there, if he has a free hand and no examinations to bother him; but I do not believe that it would work as a system.

How are mathematical teachers to come into touch with the physical sciences? This is a problem that each school has to work out for itself. One can only suggest an ideal. Take the case of a Cambridge mathematician who sets out on his campaign as a teacher with a full magazine of Mathematics, and a mathematical tripos knowledge of Physics, Mechanics, Hydrostatics, and Optics. Possibly his knowledge of Physics is entirely of the tripos type. He has possibly never experimented on the efficiency of a screw-jack or the electrical resistance of a wire. If this is the case, his outlook on Physics is all wrong; he will have to begin again. How is he to begin again?

One thing is pretty clear. To teach Mechanics in the ordinary way in a mathematical class-room would probably only confirm him in his bad habits. By the ordinary system in schools, Mechanics is one of two kinds—theoretical, belonging to the mathematician; and experimental, belonging to the science master: a wrong arrangement for all purposes.

I would have Mechanics, Hydrostatics, and Optics, apparatus and all, handed over bodily to the mathematician. (Hear, hear.)

I think, perhaps, it would be useful if I were to speak briefly in detail of the three subjects I have chosen. Mechanics appears in the mathematical course of every school, and also, I take it, in the physical course. Hydrostatics is, I think, universally taught in the physics department—barometers, pressures, and so forth. This branch does not necessarily appear in the mathematical curriculum. It is taught for certain examinations to more advanced boys. Well, if Hydrostatics is to be taught, it appears to me that we must recognise it as simply a kind of mechanics, and that there is no essential distinction between the mechanics of solids and the mechanics of fluids; and if the mathematical teacher is to teach Mechanics, he should teach Hydrostatics too. Perhaps Optics is not very generally taught; if it is to be taught, it would perhaps, by the precision of its results, and by the fact that it is not intimately linked with the other branches of physical science, appear to be a suitable subject for the mathematical department.

Let the mathematical master be made responsible for all the teaching in these subjects, experimental and theoretical. There is no educator to equal responsibility; without responsibility the ordinary man will not face the effort needed to master experimental methods of teaching. If he is made responsible, a competent mathematician will be both able and willing to learn the detail of the new work and to base his teaching on the machine, instead of the text-book. The annexation of this territory would have a very beneficial influence on the reality of the mathematical work: it borders on the introductory course of measuring and weighing, which is now an accepted part of the

mathematician's province. My experience leads me to suppose that science masters would welcome the arrangement. They would gain time for the subjects that must necessarily be theirs; and they would gain, no less than the mathematician, by the mutual understanding and sympathy that will arise when the teaching of Applied Mathematics is based on experiment.

Unless some steps of this kind can be taken to give a new point of view to the mathematical master, I fear there will be disappointment for those who hope that the recommendations of the Mathematical Association will bring on the millenium. A conscientious attempt to carry out recommendations will not take a man far on the right road, unless he is in sympathy with the motives at work when the recommendations were drafted.

I would respectfully ask the meeting to consider the definite proposals I am making, They are not revolutionary. I dare say that in many schools the arrangement that I have advocated is already in force; but it is not so, I believe, in the ordinary public school. There are certain subjects which form the borderland between Physics and Mathematics. These at present suffer from dual control; I am proposing a rectification of frontier. The advantages suggested are three: first, single control would make for good teaching; secondly, the experimental work would be good for the mathematical master; thirdly, the science master would be free to do work which is his own monopoly.

Mr. JACKSON: I should like to begin by saying that I differ from Prof. Perry on certain points, but I think we must to-day accept his first principle, namely, that we are dealing with the average boy. The whole discussion must be based on that. As I regard the retention of Mathematics as an essential element in a liberal education for the average boy as being largely due to Prof. Perry, I therefore would rather dwell on the points of principle on which I agree with him, than on the details on which I differ from him. At the same time, I do not agree with his low opinion of deductive geometry. One could mention dozens of cases in which some geometrical problem has all the interest and all the valuable features which he demands. What is the matter with the geometry of the sextant for instance?

After all, we have really got to consider what our object in education is. Emerson said that "Education is what remains when everything you learned at school is forgotten." Now, what is our object in education, and why is elementary science going to do all that is expected of it? Mr. Searle, in his book on experimental elasticity just published, observes: "A demonstrator in physics spends much of his time in correcting student's

mistakes. He has to discover, for instance, why a student obtains 537·86402 (no units stated) for Young's modulus for a brass wire. . . . The student has confused radius with diameter: has used a screw gauge in which one turn is equivalent to $\frac{1}{50}$ inch, and treated one turn as equivalent to ·5 millimetre . . . has treated the millimetres as if they were centimetres . . . and has used 32 for 'gravity' instead of 981." There is the enemy! The real enemy we have to fight against, whatever we teach, is carelessness, inaccuracy, forgetfulness, and slovenliness. That battle has been fought and won with diverse weapons. It has, for instance, been fought with Latin grammar before now and won. I say that, because we must be very careful to guard against the notion that there is any one panacea for this sort of thing. It borders on quackery to say that elementary physics will cure everything. That is why I was particularly glad to hear Prof. Perry touch on what I consider to be the real primary point. The personality and learning of the teacher are everything. Nothing else matters very much. That ought to be reiterated by every educational meeting that is held.

What is really meant by the term Practical Mathematics? I am sure Prof. Perry's meaning has been misunderstood. For instance, in the early days of the movement, he set a question about the relation between the capacity of a saucepan and its price. Some gentleman who took no interest in saucepans (I suppose he was a bachelor), took great exception to this question, and inferred that "Practical Mathematics" was a tin-pot business.

Now, what *does* Practical Mathematics mean? I doubt if Prof. Perry would be prepared to name a theorem that is certainly not "practical." If I understand him rightly, he means the term to apply as in "practical politics." You must consider not only what is right in the abstract, but what is right under given conditions. The fundamental theorem of Practical Mathematics is this: There is not only the mathematical proposition, but there is the boy into whose head that proposition is to be put, and the spirit in which the boy receives the proposition. Prof. Perry's argument is that the mathematician is apt to deal with the wrong half of the problem. He worries himself about Euclid I. 47, or about Fourier's Theorem. There is nothing wrong with them. The trouble is with the pupil who is to receive them.

I think on an occasion like this it may be more useful to mention one or two specific points than to discuss generalities. In regard to Graphs, I think the reason why they are done so badly (and I have seen a good many bad ones recently) is the kind of blackboard which is in general use. If a good graph is

never shown on the board, you cannot expect a boy to do one well. A good plan is to have one-inch squares scratched on the blackboard with a graver. The squares are quite invisible at a little distance, but they guide the hand and enable graphs to be drawn to scale, and the scales to be shown. The lightning caricaturist has his caricature drawn in faint lines, which the audience do not see. Then he goes over them with chalk. (Laughter.) This, however, is not what I recommend.

I am not quite in agreement with Prof. Perry in the view that he holds regarding the solution of triangles. It seems to me that the solution, if approached in the proper way, is as good an illustration of mathematical precision and of the common sense of mathematical 'finish' as we can have. A boy wants to find out something about a triangle. He draws it and measures its angles. Good, but first there *are* occasions when the accuracy of drawing is not enough, and besides, it is always well to be sure you are right, so he does it again by calculation of right-angled triangles—perhaps rather clumsily and at great length—but still he understands what he is doing. Now, if you tell him that there may be five hundred of such triangles to be solved, and that therefore it will pay him to find out the very shortest way of doing them, he will see the use of trigonometrical formulae. Point out, in this connection, the history of the great French Logarithmic Tables which were calculated in 1791. Owing to Republican simplicity rendering hair-powder unfashionable the hairdressers of Paris were thrown out of employment, and it is a fact that (partly through the work of Prony, the great engineer) the details of the calculations of the Tables were brought to so precise a form that the actual calculating work could be and was carried out by the unemployed hairdressers.

I do not wish to be profane, but a friend of mine once put the essence of Prof. Perry's argument in a sentence: "We want to make boys believe that Mathematics is indispensable in their daily life, and not something which they will have to do in hell." (Applause.)

Mr. ALFRED LODGE said: I think we all agree with Mr. Jackson that what we want is to make the mathematical work of the boys feel more real to them, and that it really belongs to actual life. I suppose that that is the way in which Prof. Perry, by attaching the study to scientific work, wants them to look at it too. Of course, in ordinary life there is not very much mathematics wanted, and therefore we have to go into scientific work in order to get the practice corresponding to the ability so far obtained in mathematics. I suppose that in the higher forms of a school the boys have sufficient imagination to carry on their higher mathematics without a great amount of experimental

work, if they are going to be pure mathematicians, and going to the universities as mathematical scholars; but in the lower forms I believe that a course of experimental mathematics might be most inspiring. I do not think that the whole of the class-work should be experimental; the class should be led up to the attainment of sufficient mathematical knowledge to be able to understand, and to work out correctly, the details connected with a set of experiments, such experiments being so devised as to require elucidation mathematically. This would require a great deal of preliminary arrangement, but its reward would be an added stimulus to the class to attain the mathematical knowledge required, and would give interest and purpose to their mathematical work. At present it seems that the chief stimulus is the desire to pass examinations and to obtain marks, in addition, of course, to the pleasure in work well done.

Now, most mathematical teachers have very little experience in experimental work, and would be glad of help in devising suitable experiments. But it must be remembered that what we mathematical masters want are experiments carefully graduated as illustrations of mathematical work, an entirely different object from that in the minds of science masters, whose chief aim is to teach some branch of applied science, whereas ours is to teach mathematical skill; and experiments are chiefly desirable by us in proportion as they require that skill. A very useful work on the part of the Committee would be the suggestion of a number of such experiments or references to books from which they could be obtained, such experiments being so much the better if they also lay a good foundation for scientific knowledge. Then at each school correlation could be obtained by a joint agreement between the mathematical and the science masters as to this experimental work.

Another, and probably in many cases a better way, would be for the experimental work to be conducted by the science masters. In that case they would request the mathematical masters to teach during one term the mathematics which they would require in the following term, and they would furnish the mathematical masters with some outline of their requirements and of the experiments they would be conducting. By this means a co-ordinated system would be gradually evolved and improved upon as time went on. The mathematical masters would not have actually to conduct experiments, but would teach the mathematics required for the experimental work towards the apprehension of which the class was working. In higher mathematical work the mathematical master would go on as he does now, training the students for scholarships, or whatever mathematical work they require. In these stages no difficulty arises from want of apprehension on the part of the

pupils; this happens only in the earlier stages, and where there is absence of mathematical instinct. Of course, most of these specialists would attend scientific courses, but these would be quite distinct, as their mathematics would be more than sufficient for any science work they would do in schools. If in science manuals each chapter or series of chapters were prefaced by an analysis of the mathematical knowledge required for its proper understanding, it would often be of great assistance to students. The plan I am suggesting of the science masters asking for similar work to be done term by term by the mathematical masters is on the same lines, and would give an impetus to the mathematical work which would be most useful and inspiring both to the masters and to the pupils. (Applause.)

Prof. BRYAN: Mr. W. J. Dobbs will speak from the practical as well as the mathematical side.

Mr. W. J. DOBBS: I am sure that no teacher of Elementary Mathematics can have listened to Prof. Perry's address without being stimulated by it, and I, for one, feel very grateful to him for the influence which, for some years past, he has unconsciously exercised over my own teaching. But while I agree, to a very great extent, with the general trend of his remarks, and feel grateful to him for the many stimulating and interesting suggestions which he has made, I cannot but think that, in some respects, he carries his advice a little too far. In particular, I think it would be a very great pity if deductive geometry were cut out of the usual school curriculum. I maintain that a boy has missed a most valuable part of his elementary education if he cannot appreciate the nature of an argument deducing from the supposition that A is B, that it follows as a necessary consequence that P is Q; and I hope that mathematicians will continue to influence examiners to look out for and test such qualities in their examinations. Prof. Perry seems to suggest that he feels a boy needs some such training when he advocates the writing out of a clear account of practical work done and of the conclusions arrived at. I would go further, and say that exercises in *practical geometry* should never be accepted unless they are accompanied by a concise but clear statement of the constructions made, and of the reasons for making them. I am glad to see that Prof. Perry gives a very prominent place to the subject of Mechanics, in my opinion one of the most valuable which can be included in a school curriculum, and one which, as a rule, is not introduced early enough. In this subject, at any rate, science teaching and mathematical teaching stand on common ground, and long before I introduced practical work in teaching Mechanics, I found that it enabled me to teach more Trigonometry indirectly than I was able to teach directly; and the inclusion of practical exercises in teaching that subject has

not interfered with its usefulness in this respect. I was glad to notice some remarks which were made with reference to the excessive use of formulae. How many a boy has imagined that Dynamics consists merely in substituting numerical values in such formulae as $s = ut + \frac{1}{2}ft^2$ and $P = mf$.

It is certainly true that people do not sufficiently realize the value of simple and inexpensive apparatus. I may be wrong, but I have an impression that science teachers are too fond of costly and elaborate apparatus. A great deal of the money expended in apparatus ought to be available for the pockets of the teachers, and then teachers and students would alike benefit; for students would learn more from the apparatus which they construct entirely themselves, or at any rate with only a little assistance. (Applause.)

Practical exercises should be designed either for the purpose of driving home some definite principle, or should require the use of such a principle for their interpretation. And as in Geometry, so in Mechanics, practice and theory ought to go hand in hand. The student should understand how one law gives rise to another, and should be able to deduce the second from the first. We cannot afford to do away with deductive reasoning in elementary education, and I even venture to doubt whether a student brought up entirely on the lines advocated by Prof. Perry would be likely to become a very good teacher of Mechanics. I have begun to apply some of these practical methods to the education of my two small boys. The elder is five years of age, and is just ten times as old as the younger. I have not yet started the younger on the study of the Calculus, but I notice that he spends much of his time in counting his fingers and his toes, and I think that augurs well for his early introduction to the decimal system. (Laughter.) The elder one has already started a course of weighing and measuring, using a simple home-made beam balance, and no weights less than a gramme, and as a rider a piece of wire weighing one gramme. He uses the apparatus at present to weigh only to the nearest decigram. Having determined the weights of two objects, he is able to predict what the combined weight ought to be, and he is delighted to put the proper weights into one pan before the two objects are introduced into the other. I think we shall soon reach subtraction and multiplication. At the same time I find that this simple piece of apparatus is all that is necessary for pupils of eighteen and nineteen years of age, as it is sensitive to a centigram. If the apparatus is, in the first instance, not quite as accurate as it is sensitive, so much the better. As soon as the student discovers this, let him ascertain the extent of the error, and make suggestions as to how it is to be corrected; and the apparatus, being inexpensive, can be put right by the student

himself without hesitation. Another very simple piece of apparatus that I have found most useful and instructive consists of a very light helical steel spring hanging from a pin fixed on the inside of a cupboard door in front of a graduated scale, which consists of a strip of squared paper. A light thread with a hook hangs from the spring, and on opening the door the apparatus is immediately ready for use. Suspend in succession several known weights and Hooke's Law is at once apparent. Suspend a piece of metal first in air and then in water, and determine its weight, its volume, and its specific gravity. Set the weight in motion vertically, and with the aid of an ordinary watch this piece of apparatus is sufficient to enable you to determine, to a high degree of accuracy, the value of the acceleration due to gravity. Vary the load, and observe how the time of vibration changes. The mathematical teacher will not be satisfied unless the student can show that this law is a necessary consequence of Hooke's Law. Set the body in motion through a given amplitude, and determine by considerations of work and energy the velocity of the weight in different assigned positions; and, regarding the apparatus as a machine, show by means of a graph how the power at which it is working changes with the position of the load. Now, place a platform in a suitable position below the weight hanging at rest. Raise the weight until the thread is slack, and by trial determine the position from which it must be let fall in order that it may just reach the platform. Estimate the work done by gravity during the fall. Estimate the work done against the spring, and compare the two. By this simple apparatus I find that about 99 per cent. of the work done by gravity is accounted for by the work done in pulling out the spring. It is interesting to give the student the little exercise of calculating as nearly as he can the time of the fall.

I am overwhelmed with awe when I contemplate the average boy of sixteen who has studied at the feet of Prof. Perry. At the age of twelve he knew all about centrifugal force—a term which, I believe, some of the members of the Mathematical Association have found so objectionable on account of some confusion of ideas commonly associated with it, that I think it was even recommended that the use of the expression should be entirely discontinued. Some time ago I was told by a young man of some scientific training, who sneered at the mathematician as a sort of glorified two-foot rule, that the forces which act on a small body whirling in a circle at the end of a string are five in number, viz., its momentum and its kinetic energy, both acting in the direction of motion; inertia, acting in the opposite direction; the centrifugal force outwards from the centre; and the tension of the string towards the centre. However, Prof. Perry's average boy, having

escaped the *Scylla* of deductive geometry, and perhaps for that very reason (Prof. PERRY. Hear, hear), was able to avoid the *Charybdis* of centrifugal force. I quite thought that I understood the Fourth Proposition of the First Book of Euclid long before I reached the age of sixteen, but I know now that I was suffering from a delusion; and this accounts for the fact that I have not yet attained to the perfection of Prof. Perry's average boy of sixteen. I envy him, and am tortured with the reflection that perhaps now, in common honesty, I ought to give up all further attempts at teaching mathematics, and try to realise the full significance of force as the rate of change of momentum. I am afraid I am getting a bit muddled as to the distinction between cause and effect. I thought I should get into a hole if I attempted to criticise. After all, the prominent thought in my mind is a feeling of gratitude to Prof. Perry for all that he has done. If the learner of Mathematics finds his work more interesting now than he did formerly; if the teacher of Mathematics finds his work more absorbing than ever; if we as teachers do not separate Algebra from Arithmetic, and conversely; if we teach as one subject Theoretical Geometry, Practical Geometrical Drawing, Mensuration, Trigonometry and Analytical Geometry; if we handle Mechanics in a less artificial way; if we have acquired more sympathy with the scientist and with the engineer, surely it is due in very great measure to the far-reaching influence of Professor Perry. (Applause.)

Mr. TUCKEY: I feel that the point I intended to make to the meeting is a small one. It is this: it does not at all follow that, because two subjects are taught by the same person, the correlation between the two is anything but extremely imperfect. Probably a good many of us feel that we do not satisfactorily correlate the teaching of Algebra and Geometry, for instance. That being so, the committee which has been formed will be doing great service to us collectively if the report answers a number of questions, the replies to which, we feel, will be useful to us in our teaching. It seems to me that in the elementary stages the chief difficulty of a satisfactory correlation of Science and Mathematics is that the teacher of science (whether he teaches one or both subjects), desires his pupils to understand how to find areas and volumes (including the area of a circle and the volume of a cylinder) at an earlier stage of his mathematical career than his mathematical teacher thinks easy or perhaps feasible. To find the area of a circle implies the idea of ratio—a difficult one for young boys to understand. We should like recommendations from the committee as to alterations in the mathematical course to make this idea easier; to impress the student with the meaning of a ratio at an

earlier stage than at present, and thereby enable him to understand the mensuration better.

I hope the committee will carefully consider and report as to the following points:

Should the "Unitary Method" be discarded altogether, or at a much earlier stage than at present?

Should that part of Geometry which has to do with equality of ratios be taught much earlier than at present? It seems absurd to deal with ratios of curved lines several years before ratios of straight lines.

Should areas and volumes be illustrated at once by weighing, and if so, should we use liquids as well as solids?

Another point is in regard to those questions which check themselves. We all have the greatest satisfaction in dealing with questions which contain an automatic check, but if we take as an instance the example given by Prof. Perry, the amount of multiplication required before the result is arrived at is such that probably the check is not reached until mistakes have been corrected. Will the committee suggest experiments in which less calculation is required before the check is reached?

I feel that many of us have questions to ask of this Joint Committee, and that it is important to make use of this opportunity of bringing to the notice of the committee the doubts which are in our minds regarding certain matters, such as those I have mentioned, so that we may hope for suggestions in their report which will be helpful to us in our teaching.

Prof. ARMSTRONG: I am very glad indeed to have the opportunity of joining in the chorus of appreciation with which Prof. Perry's efforts are being greeted. I think it is about time, however, that approbation of his work took that most sincere form of flattery which imitation is said to be. Although he has been talking a great many years and has produced a certain effect, the practical outcome of his labours, I am afraid, is not a very great one. He told us in the course of his paper that he was afraid he was not understood. I am sure that is the case. He told us also that he hesitated to produce a ritual which could be put into the hands of teachers. Those of us who know anything about higher teaching are all anxious not to stereotype teaching; yet I fear that we must run the risk and I venture to think that the time is come for him to be absolutely specific. A large number of teachers are looking for direction on matters of the order referred to by the last speaker (Mr. Tuckey). I have myself been working in the same direction as Prof. Perry but in another field and have some knowledge of the situation. Though I am often told that the method advocated by me is being followed, I almost invariably find that those who pay me

this compliment have not the faintest notion what I mean—no real understanding of my method. I know that is true also of Prof. Perry's method. The majority of teachers are incapable of understanding what he means, simply because they have never done any work of the kind themselves. His meaning is as absolutely misunderstood by them as if he had spoken in Chinese. And the fault lies not merely with the teachers but primarily with the universities. I take it that the average mathematical teacher has never received any practical training. He has gained an honours degree at the university, maybe, but that did not involve any practical training. Perhaps also after taking his mathematical degree he has taken a degree nominally in Physics; when you inquire into that you find it is only paper Physics that he studied. He has never taken his coat off and done real practical work; and until he does that he will never make any progress as a practical teacher.

I happen this week to have been considering the elementary mathematical teaching in a school. What I saw, for the most part, was so absolutely inane, so artificial and foreign to real practice, that I came away surprised that the boys got any useful ideas whatever into their heads.

I think the note of real importance struck by Prof. Perry to-day was his protest against the specialist. We have long been suffering from the specialist in every subject. To begin with Latin. He has taught Latin all day long and almost every day in the week with the object of producing a small percentage of boys to take scholarships at the universities. The same has happened in Mathematics. Then the idea got abroad that some subjects had special value in themselves—that there was a particular abstract value in Latin, in Mathematics, in Science. I venture to think that the point we are come to now is to doubt whether there be any particular value as mental training in any particular subject if it be taught apart from its applications. I do not believe there is. Moreover, our minds are so absolutely different in constitution that there is no one general subject of training which we can make use of to the neglect of others. Mr. Godfrey objects to incidental teaching. I doubt if there be any. Nearly all of us are teaching our particular subjects from the point of view of the absolute specialist and not from the standpoint of general requirements; we are teaching as if the boy or girl were going to develop into a specialist, not into an ordinary being. I think Prof. Perry's view is a right one. I do not go so far as to say that every man should teach every subject, but I agree with the general principle and think that in future we must have subjects taught much more broadly than is the case at present. We must get away from the tendency to professionalise which we have drifted into of late

years more and more. We must lay a real and broad foundation, and not aim too high. We have to bear in mind that the vast majority of boys and girls in school are not capable of being raised to any very high intellectual level, that therefore they must be taught with reference to what they are going to do in life, that it is absolutely useless to teach them all from the scholarship point of view and consequently to neglect them if they cannot rise to that level. I agree with Mr. Godfrey that the teaching should not be merely incidental and that we should have all the school work correlated. But at the present time there is no correlation even within departments in the average school. The work in Mathematics is not organised; the work in classics is not organised; nor, indeed, is that in any subject. Each individual master teaches his individual subject in his individual way, without reference to his colleagues. We must soon adopt the tactics in schools which our American friends have adopted in business. The headmaster must be a man who can go among his staff and, metaphorically if not actually, knock their heads together and insist that all work together in harmony; he must exercise the function that the manager of an American factory exercises at the present day: he insists upon his men working together to one end—it has to be done.

I venture to think that we must make our teaching much more practical, so that it may appeal to the children. Mr. Dobbs told us that he is making experiments with a child of five and is surprised to find how much can be done. I began in that way five-and-twenty years ago and I know what can be done. I have learned more from such work than from anything else. Prof. Perry said that our great difficulty was due to our being out of touch with the boys. I doubt if any average teacher be ever in touch with them. What we now need to do is to bring teachers, men and women alike, to see down to the level of their pupils and to think what the world needs.

Meetings of this kind are of great value in bringing about an exchange of opinions and it is from this point of view in particular that we are so much indebted to Prof. Perry to-day.

Dr. T. P. Nunn; Professor Armstrong has reminded us of Mr. Godfrey's wish for fuller information from Prof. Perry, particularly with regard to the question whether Mathematics should or should not be "incidental"; and he himself emphasised the need of a more explicit statement of Prof. Perry's intentions. Prof. Alfred Lodge's speech has made me conscious of this same need, for he suggested an interpretation of Prof. Perry's doctrine which is precisely opposite to the one that I have myself formed and have been accustomed to act upon. Prof. Lodge advocated as the proper form of correlation between Mathematics and

Science in the school that the Science Master should provide his mathematical colleagues with a programme of the experimental work to be taken in the following term, so that the latter might prepare his boys to deal with the experiments when they came to them. I must confess that I was somewhat concerned to notice that the illustration of his method which Prof. Perry gave (namely, the application of decimal multiplication in connection with the prediction and subsequent verification of the weight of a lump of iron) seemed to be in a line with Prof. Lodge's interpretation. I say I was concerned, because I have always thought that the cardinal position of Prof. Perry's doctrine was that the traditional mode of teaching Mathematics does not provide adequate *motives* for the work from the point of view of the boy, and that the only way to reform it is to take measures that such motives shall be supplied. It does not appear to me that the mere knowledge that mathematical processes learnt now are to be applied at a future date and in other connections than those in which they are learnt, constitutes to the boy a motive of the kind required, though no doubt it would affect in a salutary manner the teacher's selection of subject-matter. To gain the end proposed, it seems essential that the boy should have before him an interesting concrete problem for the solution of which his present mathematical knowledge is insufficient; and that the pursuit of the mathematical method or notation required for its solution should be regarded by him as a search for a necessary piece of apparatus which as yet his mathematical equipment does not contain. It is vital, therefore, that the problems which are to be the starting-points for the class's various mathematical enterprises should be presented to them at the time of the mathematical lesson. For example, the determination of density, the simple practical surveying problem, the investigation in optics or mechanics, must be transferred from the physical laboratory to the mathematical class-room. This of course implies that the Mathematics becomes, in a certain sense, "incidental," but not, I hope, in a sense or to a degree of which Mr. Godfrey would disapprove.

The plan I have in view will perhaps become clearer by the aid of an illustration drawn from the syllabus of an elementary school where we are trying to put Prof. Perry's ideas into operation. I will describe briefly our treatment of the process commonly known as the reduction of a vulgar to a decimal fraction. The problem which forms the point of departure of our teaching in this case is the problem of determining the ratio of the area of Great Britain to that of Ireland. Maps are drawn in the customary way on thin card, are cut out and are weighed. If there are twenty-four boys in the class, there will, as a rule, be twenty-four different values found for the fractional part of

the ratio. We have now two interesting and practical questions before us: How far are these various results in agreement with one another, and what is their average value? The only method known to the class of answering such questions in the case of fractional numbers is that of reduction to a common denominator —a method which, in the present instance, is obviously impracticable. It is seen, however, that if each boy's fraction could be "expanded" into tenths, hundredths, thousandths, etc., that the problem of determining the amount of agreement between them would become perfectly simple. The process of reduction is, therefore, accepted by them not as an academic exercise imposed upon them for inscrutable reasons, but as a weapon of obvious and immediate usefulness. May I ask your attention to one or two details in connection with this example? The first is that this process of reduction (or rather "expansion") is applied to real, as opposed to the artificially simplified cases of the text-book. Hence the boy does not start with the delusion that these reductions should normally "come out," and does not feel it as a grievance or an anomaly that those met with in real life do not do so. In the second place, the boy, before commencing computation, has to determine how far the reduction should be carried out; that is, how far the decimal places have physical significance in this particular problem. It will be agreed, I think, that both these aspects of the operation in question are important, and that both tend to be obscured in any treatment that does not make the mathematics instrumental to the solution of some such problem as I have described.

I am not a mathematician, and I speak with hesitation before so many brilliant students of the science; but I venture to suggest that in the history of Mathematics we have an invaluable clue to the best method of teaching Mathematics in school. Mathematical progress seems constantly to display a rhythm of three phases. In the first, the new method or notation is devised as an instrument to deal with a problem, generally of a physical character. Then comes a period during which the new machinery is perfected and developed, very largely apart from considerations of its usefulness. Finally comes the stage in which it is discovered to be an efficient instrument of mathematical description and investigation over a much wider range than the little group of problems in which it had its origin. I suggest to you that our teaching methods should exhibit something analogous to each of these three phases. Prof. Perry, as I understand him, has done mathematical teaching the inestimable service of insisting upon the absolute necessity of the first. The second phase, the phase of the development of technique, is represented more or less imperfectly by the traditional work of the mathematical class-room. It is a necessary phase, and one that has no lack of

interest for the boy if it is given the rational beginning and motive that I have described. Finally comes the phase of application. The work at this stage would, I think, be covered excellently by Prof. Lodge's suggestion.

One word more. Mr. Godfrey has raised the question of the frontier between the provinces of the teachers of Mathematics and Science. I do not think that any exact delimitation is possible, but I have found the following to be useful and sufficiently precise for practical purposes. All work in Mensuration and Physics that makes use of Mathematics only for the purposes of accurate description should be regarded as belonging to the mathematical master. But when the solution of the problem before the class requires the aid of a physical hypothesis it should not figure in the mathematical lesson. The necessary concentration of attention upon the hypothesis would make a problem of this kind unsuitable as the starting-point for the development of a mathematical method. Such problems should, then, not appear before the boy has acquired readiness in handling the mathematical methods necessary for their solution—that is, they should be postponed to the third stage in the development which those methods receive in the curriculum. Thus a boy might be led to adopt the decimal point notation and to perform the operations of addition, multiplication, etc., in connection with those problems of exact description which we name the determination of areas, volumes, and densities. These determinations are, therefore, to be regarded as part of the Mathematics course. On the other hand, the problems leading up to the enunciation of Archimedes' Principle, being problems in which the emphasis is upon the discovery and formulation of a satisfactory physical hypothesis, should be regarded as suitable material for the application of a knowledge of the manipulation of decimal fractions, acquired in other contexts, and should, therefore, find their place in the lesson course on Physics.

In calling upon Prof. Perry to sum up, Prof. BRYAN said he should like to refer to Prof. Armstrong's question as to whether there was a particular value in any one subject. He thought Prof. Perry would agree that a knowledge of practical elementary mathematics would make every butcher, baker, and candle-stick maker a very much better worker in his particular trade, and that the argument might apply equally to the apothecary and the ploughboy. As to the thief, his business required a very high knowledge of Mathematics. (Laughter.) He was sure that there was a great expenditure of money in teaching children what was least likely to be of value to them and make them efficient citizens. He thought all present should fight with a view to convincing the people of this country in the practical utility of elementary mathematics; and to see that Mathematics

are not crowded out of the school curricula by more luxurious and less useful subjects.

Prof. PERRY asked the indulgence of the meeting for not replying forthwith to the questions submitted. Some of the speeches of his critics had been carefully prepared, and he was afraid of giving answers which would be inadequate, and of which he would be ashamed when he afterwards saw them in print. First, as to despondency, I can never be despondent. In Japan, at Finsbury and Kensington, as chairman of examiners in seven subjects of the Science and Art Department, I was able to enforce reforms which had been the nearly hopeless aim of all good educationists; and at a certain critical moment, when there was unstable equilibrium, I happened to suggest that these old ideas should be generally adopted. They have been adopted very widely, and I get a great deal of undeserved credit. In nearly all Polytechnics and Technical Colleges, in all evening science classes, in many day classes, what is called "Mathematics" is really my new subject, "Practical Mathematics." Deductive Geometry has been given up, so has Geometrical Conics, and 95 per cent. of the orthodox Algebra and Trigonometry.

As for the Universities and public schools, I never expected quicker progress than what has been occurring in them since the Glasgow B.A. meeting. No one can despond when he sees such a gathering as this, and notices your enthusiasm for correlation in teaching Mathematics and Science. I have confidence that every attempt to correlate the usual school education with science must lead to a wider adoption of my methods. They are not new, but their general use is new. They are being widely adopted in foreign countries, and more especially in Germany.

I have been asked if I think some subjects to be important in themselves. Well, yes. And so does Prof. Armstrong. All boys ought to be able to write an account of what they have done, or thought or seen done. The evidence on this matter of Lord Roberts and other general officers after the South African war is most condemnatory of public school teaching for the average boy. All boys ought to be fond of reading English books; they ought to be able to compute; they ought to know something of physical science. These things I consider essential. In the primary and secondary schools, and university colleges of this country nearly all the attention is paid to what I call unessential subjects. I do not object to them so long as the essential subjects are taught well.

When I prepared my address I thought that every speaker would denounce me, and I have received only blessings. I cannot refrain from saying how grateful I feel to the mathe-

maticians and teachers of Mathematics and Physics who discussed things with me at the famous meetings at Glasgow and Johannesburg and here. There is a fairness, an anxiety to see things from each other's points of view that certainly presages success to our endeavours.

Our greatest point of difference is as to Deductive Geometry which I object to as a school subject. May I venture to ask my critics to look up a letter of mine in *Nature*, March 27th, 1902, and my address at Glasgow, honestly endeavouring to understand me. It is true, I have not taught boys for a long time. Thirty-eight to thirty-four years ago I had charge of the Physics teaching, and helped in the Mathematics teaching at Clifton College. But at colleges since, I have taught the poor products of the schools, boys of from 15 to 20, and men of all ages, and I have dealt with them in engineering works and in business. My methods have been tried on all kinds of pupils all my life. My opinion ought not to be set aside in an off-hand manner, and yet critics condemn without reading them. Mr. Dobbs even thinks that when I object to Deductive Geometry, I object to all reasoning! Of course I wish to have reasoning, but how can the average boy reason about things which he does not understand? Not only are the things abstract, but they are vector quantities of a most difficult aspect. Even Mr. Godfrey, with whom I am so anxious to agree, seems to think Geometry easy to think about. Mr. Dobbs thinks that he surely knew Euclid I. 4 before he was 16 years of age. Yes; and I thought, and I was assured by my teachers, that I knew the six books of Euclid before I was 12!

I know what can be said against the term centrifugal force, but my critics do not try to understand what can be said for it. I need not here reproduce what is given in the published Johannesburg discussion. I am in perfect agreement with Mr. Jackson except as regards these two matters.

Prof. Bryan made a very good point when he said that any boy ought to be allowed to use such an idea as

$$w/g = F/\text{acceleration}$$

if he finds it easy to work with, but that unfortunately examiners, if they do not object to it, will crystallise it and make a ritual of it, and make every boy try to use it. This is the point; the outside examiner is the enemy of all education. At Finsbury, where we had most success in teaching, all our methods were most unorthodox, and every teacher examined his own pupils.

And now, as to the real matter of this discussion. Mr. Godfrey, Mr. Siddons, and others, think me an idealist; they want to come to practical politics. And I say let us, however limited we may be in our practice, see that we have the true

ideal. The things we hope for and ought to work for are a doubling of salaries, a diminution of the size of classes, the ousting of mere specialists, and completely getting rid of the outside examiner.

It is wrong to say that as we cannot get to the height of our ideal, we ought to aim lower. Let me remind you that we never get to the height of what we aim at. Let us take what we can get, but let us always talk of our ideal, and if we are persistent, we shall yet arrive at it.

Prof. Bryan, who seems to be heartily with me in almost everything else, thinks that if the present ordinary master, with smatterings of all kinds of knowledge, attempts to teach many subjects he must fail miserably. Of course he must! But I want the present exceptional master to be common. I look forward to the time when every master will be properly paid, and be a man of good general culture. No man of common sense and culture would be so foolish, I might say so dishonest, as to do what Prof. Bryan suggests, and say, "It must be so, because it is in the book."

In the meantime, surely we ought to go upon Mr. Godfrey's lines. Re-arrange the boundary between Mathematics and Physics, and do not be too much afraid of over-lapping. Let the Joint Committee settle these boundary questions for the time; they may be better settled later, when we have had more experience, and I think that the specific views of both Mr. Godfrey and Dr. Nunn ought to have the committee's very serious consideration.

I have been asked what I mean by incidental teaching. Well, there are certain subjects in which the teaching ought to be only incidental: that is, now a thirty-minutes' lesson and occasionally another, but sometimes the lesson might be a five minutes' parenthesis or illustration. In any case the lesson comes casually into a more regular course of study. In every engineering college the teacher of Applied Mechanics and Heat, of Engineering Science in fact, always also teaches Mathematics. You may call it incidental teaching, but really there is a course of study of which Mathematics is an integral part. In the same way every teacher of Physics ought to have a course into which Mathematics and Mechanics may be said to come incidentally, but of which they are really integral parts. My own books on Applied Mechanics and Heat Engines will be found to illustrate what I say, and my book on the Calculus will illustrate the other side of the border. Whether the subjects of Mathematics and Physics are taught by one (and this I prefer) or by two men, there must be classes called Mathematical and there must be classes called Physical, but each of these subjects must be taught incidentally in the other class. The more overlapping the better. It is evident that Prof. Lodge's methods of correlation are

quite opposed to mine. My students now are usually more than eighteen years of age at the Royal College of Science. About half of them in my elementary class are National Scholars or Royal Exhibitioners. During half a session with me they have regular courses in Mathematics, Graphics, Descriptive Geometry, Mechanical Drawing, Laboratory Work, Numerical and other exercise work, etc. They are going to specialise later in Chemistry, Physics, Mining, Metallurgy, Mechanical and other kinds of engineering. Some of them are fairly well acquainted with Elliptic Functions, Ellipsoidal Harmonics, etc., and they can use their Mathematics in such studies as Thomson and Tait, Lamb's Hydrodynamics, Rayleigh's Sound, and Clerk Maxwell. These students benefit greatly in the elementary classes. Some of the students, again, are very ignorant of Mathematics, but before they come to the particular lectures I am going to speak about, even those who have attended only the very lowest mathematical class have some Calculus. [In the previous year the students attended regular courses in Chemistry, Physics, and Mathematics.]

Now, my lectures are on a mixture of Theoretical and Practical Mechanics and Applied Mechanics, including Hydraulics and Mathematical and Applied Thermodynamics and Heat Engines. I have the whole time of the students for half a session. Hundreds of times in that half session we differentiate or integrate x^n, $\sin(ax+e)$, $\cos(ax+e)$, x^{-1}, and ϵ^{ax}. When I have to do this sort of thing I may spend twenty minutes or only two minutes giving a mathematical lesson. In their five mathematical classes the students deal with all sorts of functions, but in this class on Engineering Science we have these five functions over and over again. We study them from every point of view. They come in incidentally in nearly every lecture and in nearly every laboratory investigation, and I have never yet known a student to seem bored, and certainly I myself never get bored, for the problems seem all so different, and there is such a pleasure in using the same mathematical tool for all of them. It is quite impossible for these students ever to forget the fundamental principles of the Calculus.

My assistants in charge of the five mathematical classes are left greatly to their own initiative, and as many of the students are preparing for London University examinations some of the courses are not right from my own point of view, but the fact that each student means to apply his Mathematics to scientific studies causes scientific illustrations to penetrate the whole of the teaching. The fifth class is really the highest kind of Mathematics as applied in Physical Science.

I think that most of Mr. Tuckey's questions are answered by the B.A. or M.A. reports. I am always in perfect agreement with Dr. Armstrong, and wish that people would try to under-

stand what he says. Dr. Nunn's illustration of the use of decimals is delightful. I envy him the opportunity for experiment which he now possesses. He certainly understands me much better than my usual critics.

I beg leave again to thank my critics for their good nature.

Mr. A. W. SIDDONS (Harrow), in proposing a vote of thanks to Prof. Perry, said he had probably been called upon because he was one of the mathematicians who, as Prof. Armstrong put it, have taken off their coats in the physical laboratory. He did not think Prof. Armstrong's charge against the mathematicians was quite fair. An increasing number of men at Cambridge were taking the Science Tripos after the Mathematical Tripos. Very few did so in his time; but he felt that, under the new regulations, more would be led to follow that course. He did not want to reply to the questions of other speakers, but he thought Prof. Perry's view that all masters should have their salaries doubled and their classes halved, showed that he was somewhat of an idealist. Such changes were not within the range of the practical politics of the immediate future. He remembered creating quite a turmoil when, soon after being appointed to a school, he set an examination paper which made it necessary for the boys to use four-figure logarithm tables; such a thing was unheard of; but the paper was set, and four-figure tables have been used ever since; so that some changes are being made, though Prof. Perry would sometimes lead us to suppose that mathematicians never made any changes in their methods of teaching.

He thought the greatest prospect of reform in the near future would be in the work of the committee, to which there had been several references that day. That committee was to consist of members of the Public Schools' Science Masters' Association and of the Mathematical Association; they were to consider what could be done to help the correlation of Mathematics and Science in Public Schools He was sure the recommendations would not go as far as Prof. Perry went; at any rate, he hoped they would not; but he hoped the report would propose definite changes which were really practicable; and if those changes were adopted in the near future, he felt sure Prof. Perry would feel that something had been done towards the realisation of his ideal. He was sure the meeting was very much indebted to Prof. Perry for the way in which, in season and out of season, he drove home the great necessity for a change.

Mr. ABBOTT proposed a vote of thanks to Dr. Bryan, who had come from Bangor especially to be present at that meeting.

The vote of thanks having been carried unanimously,

Dr. BRYAN replied that he had found great interest in the proceedings which had taken place.

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